Multi-product firms in monopolistic competition: the impact of market size

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Stylized facts about multi-product firms

- Multi-product firms account for the largest part of manufacturing output
- Intensive margins and extensive margins of firms are positively correlated:
  - Bernard, Redding and Schott (2010)
  - Goldberg et al. (2008)
  - Manova, Zhang (2011)
- There is positive correlation between firms' size and the efficiency of their R&D projects:
  - Henderson, Cockburn (1996)
  - Cockburn, Henderson (2001)

Related literature

- Ottaviano and Thisse (1999)
- Allanson and Montagna (2005)
- Johnson and Myatt (2003, 2006)
- Anderson and de Palma (2006)
- Feenstra and Ma (2007)
- Eckel and Neary (2010)
- Mayer, Melitz, Ottaviano (2011)
- Nocke and Yeaple (2012)

Questions we want to answer

- Do larger markets lead to more product diversity at the firm- and market-levels?
- How the market outcome depends on the the supply side characteristics?
- Are wider product ranges associated with lower outputs?

Why monopolistic competition

Oligopoly models:

- are difficult to handle
- neglect income effect

We propose a **more flexible** model of monopolistic competition that **mimics** oligopolistic competition.

Plan

Commodities and market structure

- There is **one sector**
- Within this sector, a **continuum** of firms of measure N operates
- Product is assumed to be horizontally differentiated across firms as well as within firms' product lines
- Each firm j chooses:
  - its **continuous** product line of size n_j
  - its production plan $\mathbf{q}_j : [0, n_j] \to \mathbb{R}_+$
- Each variety is produced by a single firm

Consumers

- The economy is endowed with L identical consumers, each of whom
  - inelastically supplies **one unit of labour**
  - maximizes her utility function

$$\mathscr{U} = \int_0^N \int_0^{n_j} u(x_{ij})\, di\, dj$$

  - and faces the budget constraint

$$\int_0^N \int_0^{n_j} p_{ij} x_{ij}\, di\, dj \leq 1$$

- The function u is assumed to be increasing, concave and thrice differentiable

Inverse demand functions

- Solving the consumer's problem yields inverse demand functions:

$$p_{ij} = \frac{u'(x_{ij})}{\lambda}$$

- λ is the marginal utility of income

- Because there is a continuum of firms, the individual influence of each firm on λ is **negligible**

Producers

- Each firm incurs:
  - a fixed cost F
  - a variable cost $V(\mathbf{q}, n)$
- The variable cost functions V is convex in $\mathbf{q}$ and satisfies the **symmetry** condition:

$$V(\mathbf{q}_1, n) = V(\mathbf{q}_2, n) \quad \forall n,$$

where $\mathbf{q}_2$ can be obtained from $\mathbf{q}_1$ by a renumbering of varieties.

Profit maximization

Because of symmetry, we can pose the firm's problem as follows:

$$\max \pi(y, n) = \frac{1}{\lambda} u'\left(\frac{y}{nL}\right) y - F - v(y, n),$$

where

- $y = \int_0^n q_i di$ is firm's total output
- v is the symmetrized cost function:

$$v(y, n) = V(\mathbf{q}, n)|_{\mathbf{q} \equiv y/n}$$

We assume v to be increasing, twice continously differentiable and convex

Examples

- Allanson and Montagna (2005):

$$v(y,n) = \phi n$$

- Feenstra and Ma (2007):

$$v(y,n) = cy + \phi n$$

- Eckel and Neary (2010), Nocke and Yeaple (2012):

$$v(y,n) = \phi n + c(n)y,$$

where ϕ stands for fixed costs per product line, whereas $c(n)$ stands for marginal production costs, which are the same for all varieties.

Scale-scope spillovers

- Empirical work finds positive correlation between the firm's size and the efficiency of its R&D projects
- So, we assume that $v(y, n)$ exhibits positive **scale-scope spillovers**:

$$v_{yn} < 0$$

Intuition

MPC decreases with respect to scope n.
Or, equivalently, marginal scope cost decrease with respect to total output y.

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Inverse demand elasticity

The **inverse demand elasticity** is given by

$$\eta(x) \equiv -\frac{xu''(x)}{u'(x)}$$

One of the key assumptions is that η is **non-decreasing** in individual consumption level x

Alternative interpretations of η:

- the reciprocal of the elasticity of substitution (under symmetric consumption pattern)
- relative love for variety

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Equilibrium conditions

Pricing:

$$p = \frac{v_y}{1 - \eta\left(\frac{y}{nL}\right)}$$

Free entry:

$$py = F + v(y, n)$$

Labour balance:

$$L = N\left(F + v(y, n)\right)$$

The "unit elasticity" condition (follows from zero profit and producer's FOC):

$$\frac{v_y\, y}{F + v(y, n)} + \frac{v_n\, n}{F + v(y, n)} = 1$$

Reduced equilibrium conditions

Lemma. *The system of equilibrium conditions can be reduced to the following system of two equations in terms of total output y and the scope n:*

$$\frac{v_y y}{F + v(y,n)} + \frac{v_n n}{F + v(y,n)} = 1$$

$$\frac{v_y y}{v_n n} = \frac{1 - \eta\left(\frac{y}{nL}\right)}{\eta\left(\frac{y}{nL}\right)}$$

Once y and n are found, the equilibrium values of price p and the mass of firms N are uniquely determined from free entry and labour balance.

Market price p^* and product diversity $n^* N^*$

Proposition 1. *Under an increase in market size L*

- *the equilibrium price p^* decreases*
- *the equilibrium output of each variety q^* increases*
- *the industry-level product diversity $n^* N^*$ increases*

Intuition

$L \uparrow \Rightarrow x \downarrow \Rightarrow \eta \downarrow \Rightarrow$ varieties become **closer substitutes** $\Rightarrow$ competitve pressure **increases** $\Rightarrow p^* \downarrow, q^* \uparrow$

Remark 1. *Changes of p^*, q^* and $n^* N^*$ are less than proportional to an increase in L.*

Remark 2. *If preferences are of CES type, both p^* and q^* remain unchanged in response to an increase in L, whereas $n^* N^*$ increases proportionally to an increase in L.*

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Firm-level product range n^*

Proposition 2. *Under an increase in market size L each firm's equilibrium scope n^* increases if and only if scale-scope spillovers exceed diseconomies of scale:*

$$n^* \uparrow \Leftrightarrow \ -n\frac{v_{yn}}{v_y} > y\frac{v_{yy}}{v_y}$$

Intuition

Market becomes larger $\Rightarrow$ firm-level product diversity **increases** (decreases) under **strong** (weak) scale-scope spillovers

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The Bernard-Redding-Schott story

We are in line with Bernard, Redding and Schott (2010) (BRS) empirical findings if

$$n^* \uparrow \quad \textbf{and} \quad q^* \uparrow$$

Can this effect be driven by an increase in market size under scale-scope spillovers?

Can we explain industry dynamics of BRS type?

Example:

$$u(x) \equiv \ln(1+x),$$

$$v(y,n) \equiv \frac{y^2}{2} + \frac{n^2}{2} - \gamma yn + \alpha y + \beta n,$$

where $\alpha, \beta \geq 0$, $0 < \gamma < 1$

Set the following parameters' values:

$$\alpha = 15.89,\ \beta = 4,\ \gamma = 0.6,\ F = 9$$

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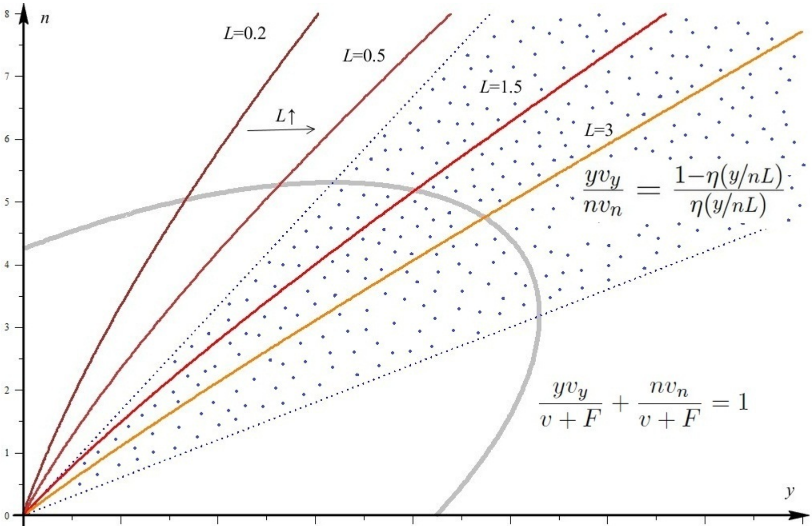
BRS or no BRS?


n
y
0
1
2
3
4
5
6
7
8
L=0.2
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L↑
$\frac{yv_y}{nv_n} = \frac{1-\eta(y/nL)}{\eta(y/nL)}$
$\frac{yv_y}{v+F} + \frac{nv_n}{v+F} = 1$

Negative spillovers

Proposition 3. *No positive spillovers – no BRS.*

Firm-level total output y^*

Proposition 4. *Under an increase in market size L the firm-level total output y^* increases if and only if diseconomies of scope exceed scale-scope spillovers:*

$$y^* \uparrow \Leftrightarrow \quad n\frac{v_{nn}}{v_n} > -y\frac{v_{ny}}{v_n}$$

Intuition

Relatively strong **diseconomies of scope** (compared to scale-scope spillovers) $\Rightarrow$ **intensive margins** increase relative to **extensive margins** $\Rightarrow \; y^* \uparrow$

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The mass of firms N^*

Proposition 4. *The mass of firms N^* increases* **more than proportionally** *in response to an increase in market size L if and only if in equilibrium net diseconomies of scale exceed net diseconomies of scope:*

$$\frac{\partial \ln N^*}{\partial \ln L} > 1 \Leftrightarrow y\frac{v_{yy}}{v_y} + n\frac{v_{yn}}{v_y} > n\frac{v_{nn}}{v_n} + y\frac{v_{ny}}{v_n}$$

Intuition

Low impact of scope expansions in reducing MPC $\Rightarrow$ **more firms** rather than **more varieties per firm**

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No scale-scope spillover

Each firm incurs:

- a fixed cost F
- an R&D cost (or monitoring cost) $S(n)$
- a variable production cost $v(y)$

Total cost is $F + V(y) + S(n)$.

In this case: **no 3s**.

Comparative statics under no 3s

Proposition 5. *Assume that there are no scale-scope spillovers. Then under an increase in market size L:*

- *each firm's total output y^* increases*
- *each firm's product line n^* shrinks*
- *the mass of firms N^* increases*

Intuition

$L \uparrow \Rightarrow n^* \downarrow$, $N^* \uparrow \Rightarrow$ larger markets are **less concentrated**

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Per-variety additive costs

Consider variable costs which is **additive across varieties**:

$$V(\mathbf{q}, n) = \int_0^n v(q_i)di + S(n),$$

where

- v is a variable cost of a separate plant/variety
- S is an R&D cost (or monitoring cost)

Comparative statics under per-variety additive costs

Proposition 10. *Assume that production costs are additive across varieties. Then under an increase in market size L:*

- *scopes n^* remain unchanged*
- *the mass of firms N^* increases*

Intuition

The manager of each variety maximizes profits **for her variety** regardless of what the others do $\Rightarrow$ everything works as if each firm is were **a set of single-product firms**

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Future work

- Heterogeneous firms
- Open economy with trade costs
- Firms' endogenous choice between being single- or multi-product

Thank you for your attention!